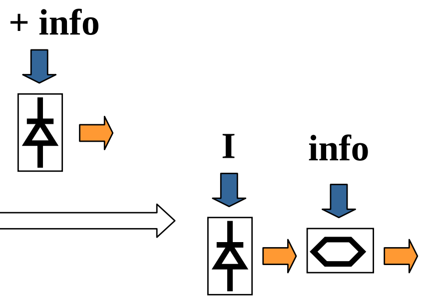
MODULACIÓ DEL LÀSER

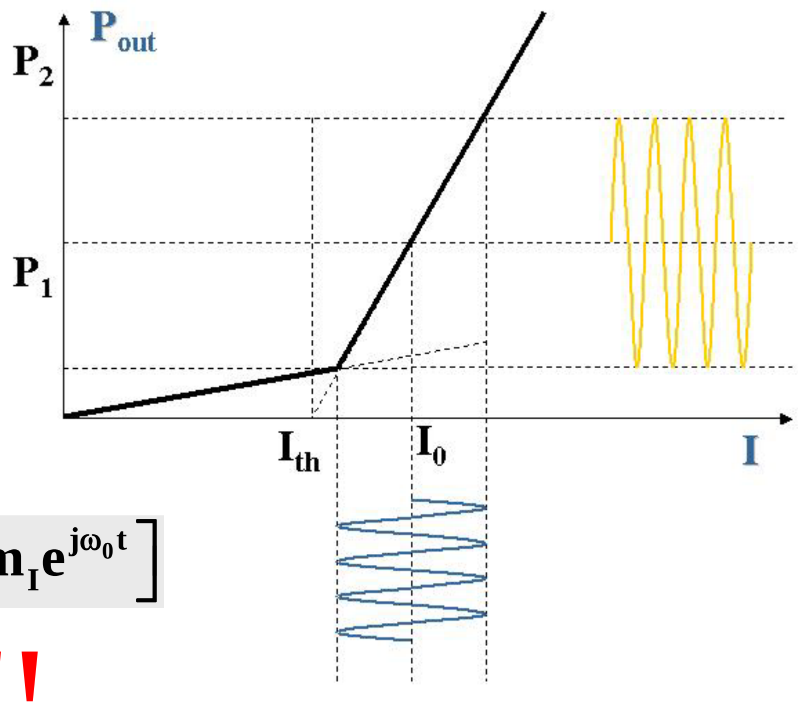
Tipus de Modulació

- **Modulació d'Intensitat** ⇒ potència òptica (IM)
- **Modulació Coherent** (portadora òptica) ⇒ amplitud (ASK), freqüència (FSK), fase (PSK)
- **Modulació Directa** ⇒
- **Modulació Externa** ⇒



COMUNICACIONS ÒPTIQUES

Modulació Sinusoïdal



$$I(t) \equiv I_0\left[1 + m_I e^{j\omega_0 t}\right]$$

$m_I << 1$

Petita senyal **!!**

$$I(t) \equiv I_0\left[1 + m_J e^{j\omega_0 t}\right] \equiv I_0 + \Delta I(t)$$ Corrent de modulació

Règim estacionari

Comportament Estàtic

$$S = \frac{\tau_p}{qV}\left(I - I_{th}\right)$$
$$N = N_{th}$$

$$\omega_0 \to 0$$
$$I \equiv I_0\left[1 + m_J\right]$$
$$S = \frac{\tau_p}{qV}\left(I - I_{th}\right) = \underbrace{\frac{\tau_p}{qV}\left(I_0 - I_{th}\right)}_{S_0} + \underbrace{\frac{\tau_p}{qV} m_J I_0}_{\Delta S(0)}$$

Règim sinusoïdal

$$S \equiv S_0 + \Delta S(t)$$
$$N \equiv N_{th} + \Delta N(t)$$

Variació sinusoïdal

S_0: densitat de fotons de la component contínua
N_{th}: densitat de portadors de la component contínua

Equació de Ritme de Portadors

$$S \equiv S_0 + \Delta S(t)$$
$$N \equiv N_{th} + \Delta N(t)$$
$$g|_{\lambda_p} = \Gamma a\left(N - N_0\right)$$

$$\frac{\partial N}{\partial t} = \frac{I}{qV} - \frac{N}{\tau_r} - vgS \quad \leftarrow g(N) = g\left(N_{th} + \Delta N\right) = \underbrace{\Gamma a\left(N_{th} - N_0\right)}_{\alpha_t} + \Gamma a \cdot \Delta N$$

$$\frac{\partial \Delta N}{\partial t} = \frac{I_0 + \Delta I}{qV} - \frac{N_{th} + \Delta N}{\tau_r} - v\left[\alpha_t + \Gamma a \cdot \Delta N\right]\cdot\left(S_0 + \Delta S\right)$$

$$N_{th} = N_0 + \frac{\alpha_t}{\Gamma a}$$

$$N_{th} = I_{th}\frac{\tau_r}{qV} \qquad S_0 = \frac{\tau_p}{qV}\left(I_0 - I_{th}\right)$$

$$\frac{\partial \Delta N}{\partial t} = \underbrace{\frac{I_0 - I_{th}}{qV}}_{S_0/\tau_p} + \frac{\Delta I}{qV} - \frac{\Delta N}{\tau_r} - v\left[\alpha_t + \Gamma a \cdot \Delta N\right]\cdot\left(S_0 + \Delta S\right)$$

$$\tau_p \equiv \frac{1}{v \cdot \alpha_t}$$

$$\frac{\partial \Delta N}{\partial t} = \frac{S_0}{\tau_p} + \frac{\Delta I}{qV} - \frac{\Delta N}{\tau_r} - \left[\frac{S_0}{\tau_p} + \frac{\Delta S}{\tau_p} + v\Gamma a \cdot \Delta N \cdot S_0 + \underbrace{v\Gamma a \cdot \Delta N \cdot \Delta S}_{\text{neglible}}\right]$$

$$\boxed{\frac{\partial \Delta N}{\partial t} = -\Delta N\left(\frac{1}{\tau_r} + v\cdot\Gamma a\cdot S_0\right) - \frac{\Delta S}{\tau_p} + \frac{\Delta I}{qV}} \quad (1) \quad \xrightarrow{\;\frac{\partial}{\partial t}\;}$$

Equació de Ritme de Fotons

$$S \equiv S_0 + \Delta S(t) \qquad \tau_p \equiv \frac{1}{v\cdot\alpha_t}$$

$$g = \alpha_t + \Gamma a\cdot\Delta N$$

$$\frac{\partial S}{\partial t} = v\cdot g\cdot S - v\cdot\alpha_t S + \cancel{\beta\frac{N}{\tau_r}} \approx \frac{S}{\tau_p}\left(\frac{g}{\alpha_t} - 1\right)$$

$$\frac{\partial \Delta S}{\partial t} = \frac{(S_0+\Delta S)}{\tau_p}\left(\frac{\cancel{\alpha_t} + \Gamma a\cdot\Delta N}{\alpha_t} - \cancel{1}\right) = v\Gamma a\cdot S_0\Delta N + \cancel{\underbrace{v\Gamma a\cdot\Delta S\cdot\Delta N}_{\text{negligible}}}$$

$$\boxed{\frac{\partial \Delta S}{\partial t} \approx v\Gamma a\cdot S_0\Delta N} \quad (2) \quad \longrightarrow$$

Derivant (1) i substituint en (2)

$$\frac{\partial^2 \Delta N}{\partial t^2} + \frac{\partial \Delta N}{\partial t}\left(\frac{1}{\tau_r} + v\Gamma a\cdot S_0\right) + \frac{v\Gamma a\cdot S_0}{\tau_p}\Delta N = \frac{1}{qV}\frac{\partial \Delta I}{\partial t}$$

α: constant de decaïment del làser → $\sim 10^9$

ω_c: freqüència de ressonància del làser → $\sim 10^{12}$

$$2\alpha \equiv \frac{1}{\tau_r} + v\Gamma a\cdot S_0$$

$$\omega_c^2 \equiv \frac{v\Gamma a\cdot S_0}{\tau_p}$$

Equació de Variació de Portadors

$$\boxed{\frac{\partial^2 \Delta N(t)}{\partial t^2} + 2\alpha\frac{\partial \Delta N(t)}{\partial t} + \omega_c^2\Delta N(t) = \frac{1}{qV}\frac{\partial \Delta I(t)}{\partial t}}$$

$$\alpha \ll \omega_0$$

Oscil·lació Sinusoidal

$$\frac{\partial^2 \Delta N(t)}{\partial t^2} + 2\alpha \frac{\partial \Delta N(t)}{\partial t} + \omega_c^2 \Delta N(t) = \frac{1}{qV}\frac{\partial \Delta I(t)}{\partial t}$$

$$\Delta I(t) = m_I I_0 e^{j\omega_0 t} u(t)$$
$$\updownarrow$$
$$\Delta I(\omega) = m_I I_0 \left[\pi\partial\left(\omega-\omega_0\right) + \frac{1}{j\left(\omega-\omega_0\right)} \right]$$

TF ↓

$$-\omega^2 \Delta N(\omega) + j2\alpha\omega\Delta N(\omega) + \omega_c^2 \Delta N(\omega) = \frac{1}{qV} j\omega \Delta I(\omega)$$

$$\Delta N(\omega)\Big[\underbrace{\omega_c^2 + j2\alpha\omega - \omega^2}_{\omega_c^2 - \alpha^2 + (\alpha + j\omega)^2}\Big] = \frac{1}{qV} m_I I_0 j\omega \left[\pi\partial\left(\omega-\omega_0\right) + \frac{1}{j\left(\omega-\omega_0\right)} \right]$$

$$\Delta N(\omega) = \frac{m_I I_0}{qV} j\omega \left[\pi\partial\left(\omega-\omega_0\right) + \frac{1}{j\left(\omega-\omega_0\right)} \right] \frac{1}{\Omega^2 + (\alpha + j\omega)^2}$$

$$\Delta N(\omega) = \frac{m_I I_0}{qV} j\omega \left[\underbrace{\pi\partial\left(\omega-\omega_0\right)\frac{1}{\Omega^2 + (\alpha + j\omega)^2}}_{\partial(\omega-\omega_0)\frac{1}{\Omega^2+(\alpha+j\omega_0)^2}} + \underbrace{\frac{1}{j\left(\omega-\omega_0\right)}\frac{1}{\Omega^2 + (\alpha + j\omega)^2}}_{\left[\frac{1}{j(\omega-\omega_0)} - \frac{(\alpha+j\omega_0)+(\alpha+j\omega)}{\Omega^2+(\alpha+j\omega)^2}\right]\frac{1}{\Omega^2+(\alpha+j\omega_0)^2}} \right]$$

$$\Delta N(\omega) = \frac{m_I I_0}{qV} \frac{j\omega}{\Omega^2 + (\alpha + j\omega_0)^2} \left[\underbrace{\pi\partial\left(\omega-\omega_0\right) + \frac{1}{j\left(\omega-\omega_0\right)}}_{\frac{1}{2\pi}e^{j\omega_0 t}u(t)} - \underbrace{\frac{(\alpha+j\omega_0)+(\alpha+j\omega)}{\Omega^2 + (\alpha + j\omega)^2}}_{\frac{\alpha+j\omega_0}{\Omega^2+(\alpha+j\omega)^2} + \frac{\alpha+j\omega}{\Omega^2+(\alpha+j\omega)^2}} \right]$$

TF^{-1} ↓

$$-\frac{\alpha + j\omega_0}{\Omega}\underbrace{\frac{\Omega}{\Omega^2 + (\alpha + j\omega)^2}}_{e^{-\alpha t}\sin(\Omega t)u(t)} - \underbrace{\frac{\alpha + j\omega}{\Omega^2 + (\alpha + j\omega)^2}}_{e^{-\alpha t}\cos(\Omega t)u(t)}$$

$$\Delta N(t) = \frac{m_I I_0}{qV} \frac{1}{\Omega^2 + (\alpha + j\omega_0)^2} \frac{\partial}{\partial t}\left\{ e^{j\omega_0 t}u(t) - \frac{\alpha + j\omega_0}{\Omega} e^{-\alpha t}\sin(\Omega t)u(t) - \right.$$

$$\left. - e^{-\alpha t}\cos(\Omega t)u(t) \right\}$$

$$\frac{\partial}{\partial t}\left\{e^{j\omega_0 t}u(t)\right\} = j\omega_0 e^{j\omega_0 t}u(t) + e^{j\omega_0 t}\partial(t) = j\omega_0 e^{j\omega_0 t}u(t) + \partial(t)$$

$$\frac{\partial}{\partial t}\left\{\frac{\alpha + j\omega_0}{\Omega} e^{-\alpha t}\sin(\Omega t)u(t)\right\} = \left[-\alpha\frac{\alpha + j\omega_0}{\Omega} e^{-\alpha t}\sin(\Omega t) + (\not{\alpha} + j\omega_0)e^{-\alpha t}\cos(\Omega t)\right]u(t) + \underbrace{\frac{\alpha + j\omega_0}{\Omega} e^{-\alpha t}\sin(\Omega t)\partial(t)}_{0}$$

$$\frac{\partial}{\partial t}\left\{e^{-\alpha t}\cos(\Omega t)u(t)\right\} = \left[\cancel{-\alpha e^{-\alpha t}\cos(\Omega t)} - \Omega e^{-\alpha t}\sin(\Omega t)\right]u(t) + \underbrace{e^{-\alpha t}\cos(\Omega t)\partial(t)}_{\partial(t)}$$

$$\Delta N(t) = \frac{m_I I_0}{qV} \frac{1}{\Omega^2 + (\alpha + j\omega_0)^2} \left\{ \begin{array}{l} j\omega_0 e^{j\omega_0 t}u(t) + \cancel{\partial(t)} - \cancel{\partial(t)} + \\ + \left[\begin{array}{l} \alpha\frac{\alpha + j\omega_0}{\Omega} e^{-\alpha t}\sin(\Omega t) - j\omega_0 e^{-\alpha t}\cos(\Omega t) \\ \qquad + \Omega e^{-\alpha t}\sin(\Omega t) \end{array} \right] u(t) \end{array} \right\}$$

$$\Delta N(t) = \frac{m_I I_0}{qV} \frac{1}{\Omega^2 + (\alpha + j\omega_0)^2} \left\{ j\omega_0 e^{j\omega_0 t} + \cancel{\left[\alpha\frac{\alpha + j\omega_0}{\Omega} + \Omega\right] e^{-\alpha t}\sin(\Omega t)} - \right.$$

$$\left. \cancel{- j\omega_0 e^{-\alpha t}\cos(\Omega t)} \right\} u(t)$$

En Règim Permanent

$$\left.\Delta N(t)\right|_{t\to\infty} = \frac{m_I I_0}{qV} \frac{j\omega_0}{\Omega^2 + (\alpha + j\omega_0)^2} e^{j\omega_0 t} = \frac{I_0}{qV} m_I \frac{j\omega_0}{\omega_c^2 - \omega_0^2 + j2\alpha\omega_0} e^{j\omega_0 t}$$

$$\Omega^2 \equiv \omega_c^2 - \alpha^2$$

Portadors

Fotons

$$\Delta N(t)=\frac{m_I I_0}{qV}\frac{1}{\omega_c^2-\omega_0^2+j2\alpha\omega_0}\frac{\partial}{\partial t}\left\{\left[e^{j\omega_0 t}-\frac{\alpha+j\omega_0}{\Omega}e^{-\alpha t}\sin(\Omega t)-e^{-\alpha t}\cos(\Omega t)\right]u(t)\right\}$$

$$\frac{\partial \Delta S(t)}{\partial t}=v\Gamma a\cdot S_0\Delta N(t)$$

$$\Delta S(t)=v\Gamma a\cdot S_0\frac{m_I I_0}{qV}\frac{1}{\omega_c^2-\omega_0^2+j2\alpha\omega_0}\left[e^{j\omega_0 t}-\frac{\alpha+j\omega_0}{\Omega}e^{-\alpha t}\sin(\Omega t)-e^{-\alpha t}\cos(\Omega t)\right]u(t)$$

En Règim Permanent

$$\left.\Delta S(t)\right|_{t\to\infty}=v\Gamma a\cdot S_0\frac{I_0}{qV}\frac{m_I}{\omega_c^2-\omega_0^2+j2\alpha\omega_0}e^{j\omega_0 t}=\frac{I_0}{qV}\tau_p m_I\frac{\omega_c^2}{\omega_c^2-\omega_0^2+j2\alpha\omega_0}e^{j\omega_0 t}$$

$$\omega_c^2\equiv\frac{v\Gamma a}{\tau_p}S_0$$

Resposta a un Senyal Sinusoidal en Règim Permanent

$$I(t)=I_0\left[1+m_I e^{j\omega_0 t}\right]u(t)$$

$$N\equiv N_{th}+\Delta N(t) \qquad N_{th}=I_{th}\frac{\tau_r}{qV}$$

$$\left.N(t)\right|_{t\to\infty}=N_{th}+\frac{I_0}{qV}m_I\frac{j\omega_0}{\omega_c^2-\omega_0^2+j2\alpha\omega_0}e^{j\omega_0 t}=N_{th}\left[1+\underbrace{\frac{I_0}{I_{th}}\frac{m_I}{\tau_r}\frac{j\omega_0}{\omega_c^2-\omega_0^2+j2\alpha\omega_0}}_{m_N}e^{j\omega_0 t}\right]$$

$$S\equiv S_0+\Delta S(t)$$

$$\left.S(t)\right|_{t\to\infty}=S_0+\frac{I_0}{qV}\tau_p m_I\frac{\omega_c^2}{\omega_c^2-\omega_0^2+j2\alpha\omega_0}e^{j\omega_0 t}=S_0\left[1+\frac{I_0}{qV}\frac{\tau_p m_I}{S_0}\frac{\omega_c^2}{\omega_c^2-\omega_0^2+j2\alpha\omega_0}e^{j\omega_0 t}\right]$$

$$\left.S(t)\right|_{t\to\infty}=S_0\left[1+\underbrace{\frac{I_0}{I_0-I_{th}}m_I\frac{\omega_c^2}{\omega_c^2-\omega_0^2+j2\alpha\omega_0}}_{m_S}e^{j\omega_0 t}\right] \qquad S_0=\frac{\tau_p}{qV}\left(I_0-I_{th}\right)$$

Potència de Sortida en Règim Permanent

$$S(t)\big|_{t\to\infty} = S_0\left[1+\underbrace{\frac{I_0}{I_0-I_{th}}m_I}_{m_S}\frac{\omega_c^2}{\omega_c^2-\omega_0^2+j2\alpha\omega_0}e^{j\omega_0 t}\right]$$

$$P(t)=\frac{1-R}{2\sqrt{R}}\cdot v\cdot Wd\cdot hf\cdot S(t)$$

$$P(t)=\frac{1-R}{2\sqrt{R}}\cdot v\cdot Wd\cdot hf\cdot S_0\left[1+\underbrace{\frac{I_0}{I_0-I_{th}}m_I\frac{\omega_c^2}{\omega_c^2-\omega_0^2+j2\alpha\omega_0}}_{m_S}e^{j\omega_0 t}\right]$$

$$S_0=\frac{\tau_p}{qV}\left(I_0-I_{th}\right)$$

$$P(t)=\frac{1-R}{2\sqrt{R}}\frac{1}{\alpha_t L}\frac{hf}{q}\left[\left(I_0-I_{th}\right)+I_0 m_I\underbrace{\frac{\omega_c^2}{\omega_c^2-\omega_0^2+j2\alpha\omega_0}}_{M(\omega)}e^{j\omega t}\right]=P_0+\Delta P(t)$$

$$P_0=\frac{1-R}{2\sqrt{R}}\frac{1}{\alpha_t L}\frac{hf}{q}\left(I_0-I_{th}\right)$$

$$\Delta P(t)=\frac{1-R}{2\sqrt{R}}\frac{1}{\alpha_t L}\frac{hf}{q}I_0 m_I M(\omega)e^{j\omega t}$$

Funció de Transferència

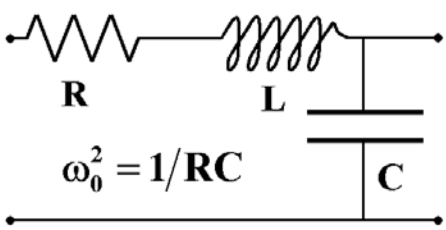
$$H(\omega)\equiv\frac{\Delta P}{\Delta I}=\frac{\frac{1-R}{2\sqrt{R}}\frac{1}{\alpha_t L}\frac{hf}{q}I_0 m_I M(\omega)e^{j\omega t}}{I_0 m_I e^{j\omega t}}=\frac{1-R}{2\sqrt{R}}\frac{1}{\alpha_t L}\frac{hf}{q}M(\omega)$$

$$\Delta P(t)=\frac{1-R}{2\sqrt{R}}\frac{1}{\alpha_t L}\frac{hf}{q}I_0 m_I M(\omega)e^{j\omega t}$$

$$\Delta I=I_0 m_I e^{j\omega t}$$

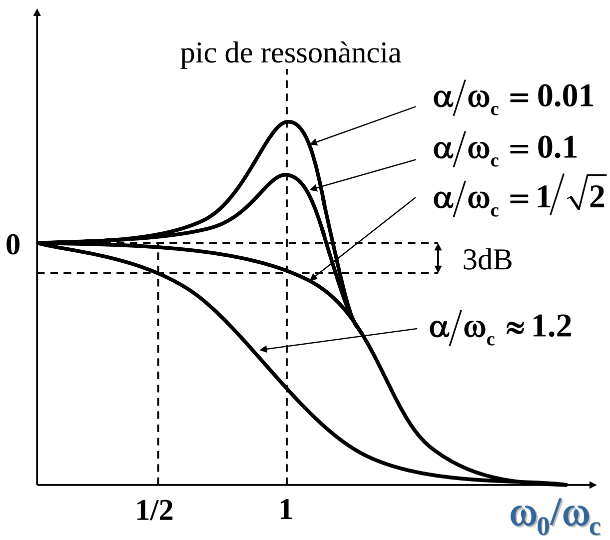
$$\overline{H(\omega)}=M(\omega)=\frac{\omega_c^2}{\omega_c^2-\omega_0^2+j2\alpha\omega_0}$$

$$\left|M(\omega)\right|^2=\frac{1}{\left[1-\left(\frac{\omega_0}{\omega_c}\right)^2\right]^2+\left[2\alpha\frac{\omega_0}{\omega_c^2}\right]^2}$$



Filtre passa-baixes de 2n. Ordre

$20\log|M(\omega)|$



Ample de Banda

$$BW\big|_{NO\_RES} = \omega_0$$

$$BW\big|_{RES} \approx \omega_0/2$$

$$|M(\omega)|^2 = \frac{1}{\left[1-\left(\frac{\omega_0}{\omega_c}\right)^2\right]^2 + \left[2\alpha\frac{\omega_0}{\omega_c^2}\right]^2}$$

$$\omega_c^2 \equiv \frac{v\Gamma a \cdot S_0}{\tau_p} \qquad 2\alpha \equiv \frac{1}{\tau_r} + v\Gamma a \cdot S_0$$

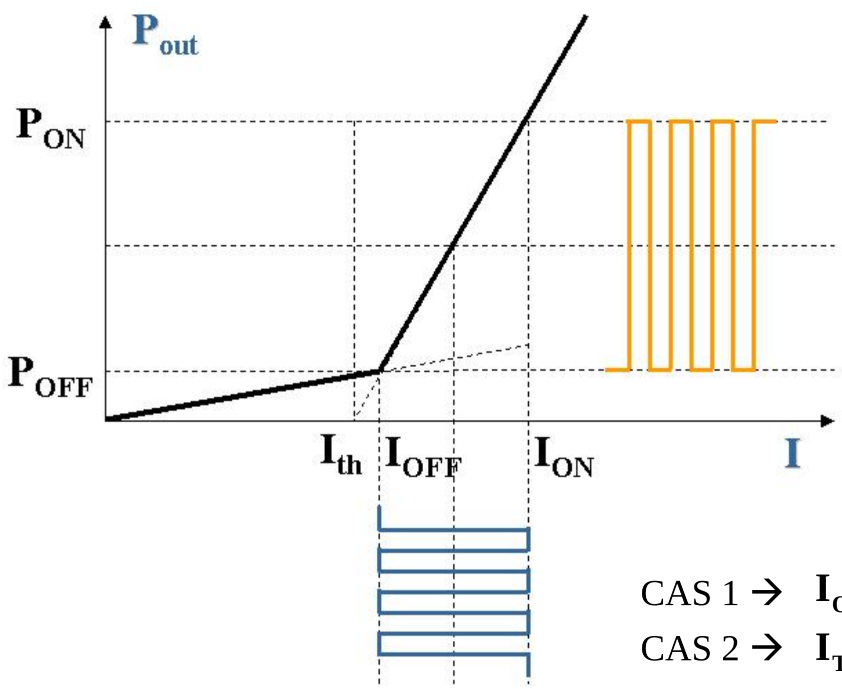
COMPROMÍS ➡

$\alpha/\omega_c < 1/2 \rightarrow$ ressonància

$\alpha/\omega_c > 1/\sqrt{2} \rightarrow BW < \omega_c$

$$S_0 = \frac{\tau_p}{qd}\left(I_0 - I_{th}\right)$$

Modulació Digital



CAS 1 → $I_{OFF} < I_{TH} < I_{ON}$

CAS 2 → $I_{TH} < I_{OFF} < I_{ON}$

Modulació Digital – Graó de Corrent

$$\Delta I(t) = I_0 u(t)$$

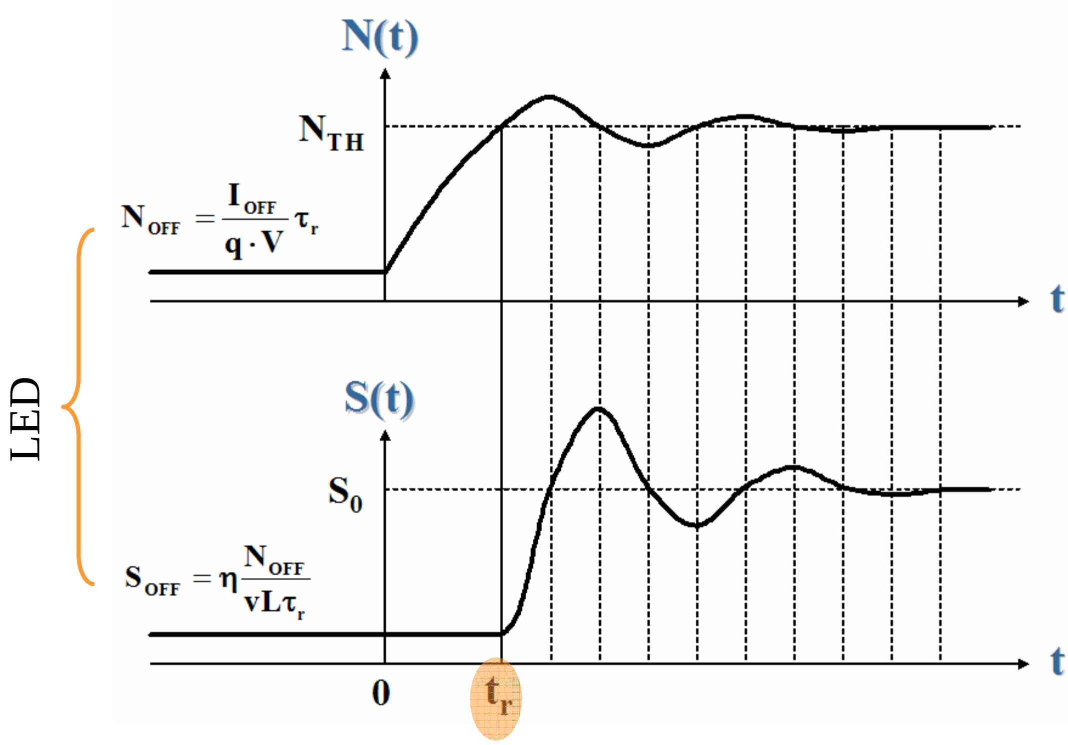
$$\Delta N(t) = \frac{I_0}{qV}\frac{1}{\omega_c^2}\left\{\left[\frac{\omega_c^2}{\sqrt{\omega_c^2-\alpha^2}}\right]e^{-\alpha t}\sin(\Omega t)\right\}u(t) \approx \frac{I_0}{qV}e^{-\alpha t}\sin(\Omega t)u(t)$$

$$\Delta S(t) = v\Gamma a \cdot S_0 \frac{I_0}{qV}\frac{1}{\omega_c^2}\left[1-\frac{\alpha}{\sqrt{\omega_c^2-\alpha^2}}e^{-\alpha t}\sin(\Omega t) - e^{-\alpha t}\cos(\Omega t)\right]u(t) \approx$$

$$\Delta S(t) \approx v\Gamma a \cdot S_0 \frac{I_0}{qV}\frac{1}{\omega_c^2}\left[1-e^{-\alpha t}\cos(\Omega t)\right]u(t)$$

Oscil·lacions Sinusoidals Esmorteïdes Desfassades 90º

$I_{OFF} < I_{TH} < I_{ON}$



Només emissió espontània (LED)

$$N(t) = \frac{\tau_r}{qV} I_{ON} + \frac{\tau_r}{qV}\left[I_{ON} - I_{OFF}\right]\left(1 - e^{-t/\tau_r}\right) \qquad 0 \le t \le t_r$$

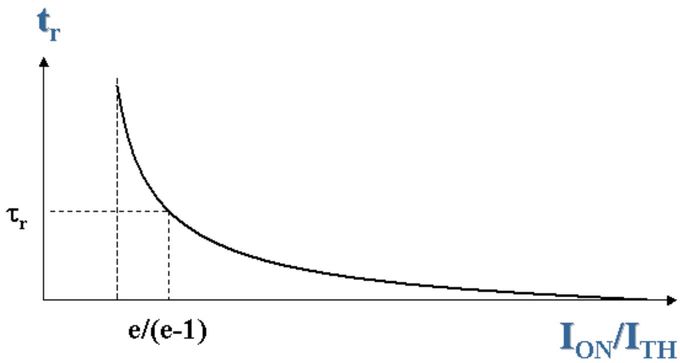
Temps de Resposta

$$t_r = \tau_r \ln \frac{I_{ON} - I_{OFF}}{I_{ON} - I_{TH}}$$

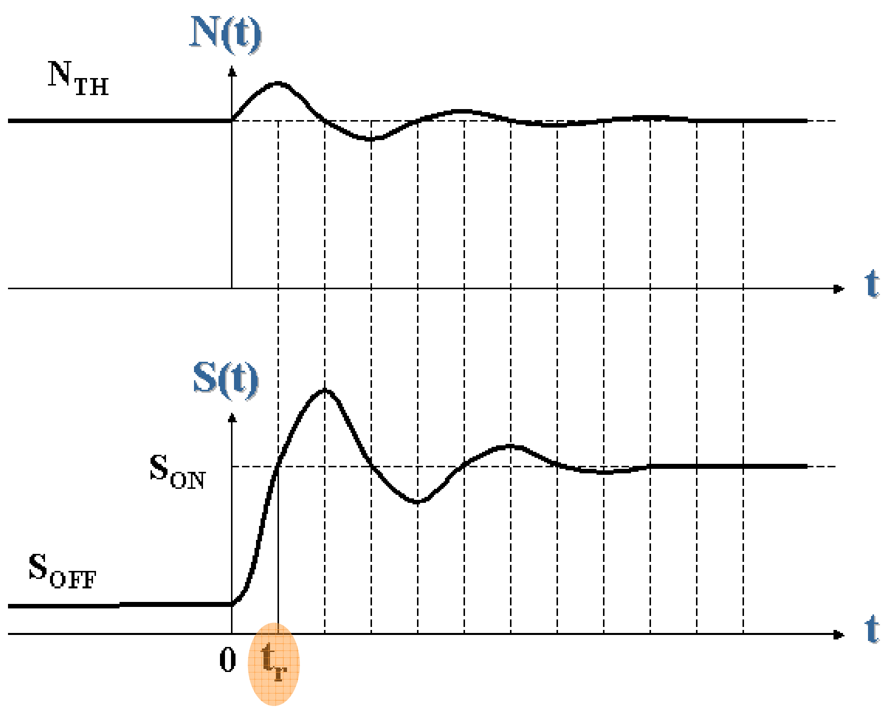
$$I_{OFF} = 0 \rightarrow \quad t_r = \tau_r \ln \frac{I_{ON}/I_{TH}}{I_{ON}/I_{TH} - 1}$$

$$t_r\big|_{LED} = 2.19\tau_r$$

En aquest cas no tenim un t_r fixat com en el LED



$$I_{TH} < I_{OFF} < I_{ON}$$



Sempre emissió estimulada (LÀSER)

$$N(t) \approx N_{TH} + \frac{I_{ON} - I_{OFF}}{q \cdot V} t \qquad 0 \le t \le t_r$$

$$S(t) \approx S_{OFF} \exp\left[\frac{v\Gamma a}{2qV}\left(I_{ON} - I_{OFF}\right)t^2\right] \qquad 0 \le t \le t_r$$

$$t_r \approx \left[\frac{2qV}{v\Gamma a} \frac{\ln\left(\left(I_{ON} - I_{TH}\right)/\left(I_{OFF} - I_{TH}\right)\right)}{I_{ON} - I_{OFF}}\right]^{1/2}$$

⟸ No depèn de τ_r

Aquest cas dóna molt millors prestacions però té un consum molt elevat. El cas anterior és just el contrari.